MODAL SPACE - IN OUR OWN LITTLE WORLD *by Pete Avitabile*

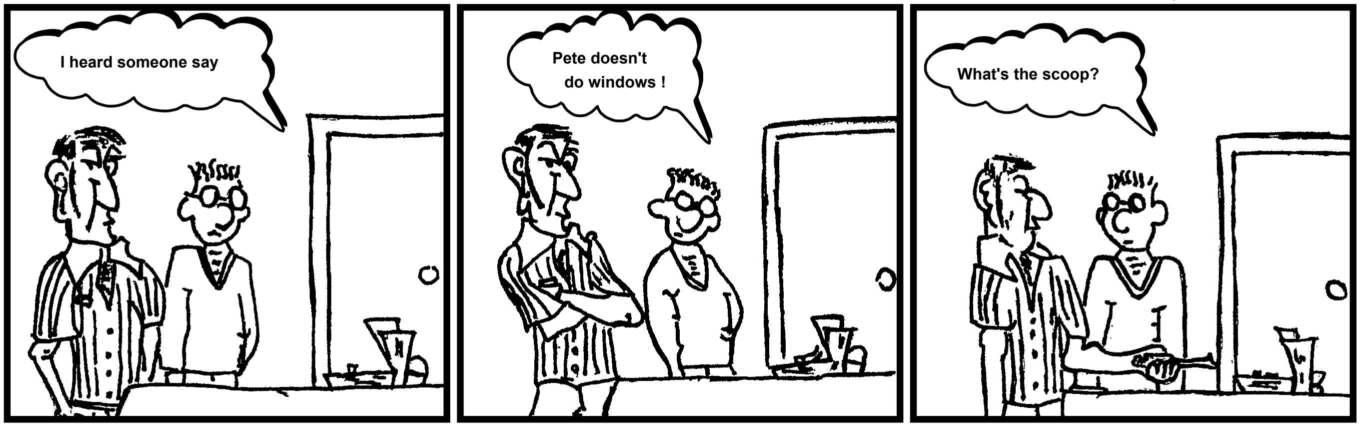

Illustration by Mike Avitabile

I heard someone say Pete doesn't do windows! What's the scoop?

Well ... that's right. But you have to let me qualify that statement. Of course there are many data acquisition situations where it is a necessity to use windows. But almost all of the time when performing a modal test, the input excitation can be selected such that the use of windows can be eliminated. Let's first understand why acquisition of certain types of data can be distorted by the digitization and sampling process, what needs to be done to minimize the distortion, and how to work around the acquisition problem through the selection of specialized test excitation techniques.

First let's remember that the Fourier Transform is defined from $-\infty$ to $+\infty$ but that we only acquire data over a very short time interval. As long as we can reconstruct the data, for all time, from the very small sample we measure, then there is no problem.

Figure 1 shows a simple sine wave, sampled for one time record, with the reconstruction of the time signal from the sample. Figure 1 also shows the FFT of this sampled signal. The time signal is expressed in the frequency domain as one discrete spectral line as expected. This happened because we captured an integer number of cycles of the sine wave in one record or sample of the data - in which case we say that the signal is periodic with respect to the sample interval.

But what if this is not the case. Figure 2 shows this situation. As before, we see the signal, the sample, the reconstructed signal and the FFT of the signal. Notice that the reconstructed signal contains a discontinuity that clearly did not exist in the original signal. The FFT of this signal is far from being a single spectral line as expected. Due to the sampling distortion, the frequency representation is smeared over the whole frequency bandwidth. This very serious error is called leakage and is by far the most serious digital signal processing error that is encountered.
But why does this happen? The original signal was a simple sine wave. How did the frequency representation get so distorted? There's an easy explanation for this. The sampled data does ***not*** contain an integer number of cycles or repetitions of the signal.

Let's stop and recall some simple things we learned about Fourier series. If we start with a simple sine wave, we know that it is a trivial task to describe that signal with a Fourier series. It is basically just one term of the Fourier series which is a sine wave at ω with some amplitude A_0. But do you remember what the series expansion was for a signal such as a rectangular series of pulses? Well, I don't want to expand on all of this right now but I think you would remember that it was a series of sinusoids at different frequencies with different amplitudes. In fact for the rectangular pulse, there were many terms in the series required in order to approximate that signal. That happened because the shape of the discontinuous rectangular pulse doesn't look like a nice smooth sine wave.

Now if I look back at the sampled sine wave in Figure 2, I can now see that by not capturing an integer number of cycles of the signal I have distorted the signal such that it *appears* to have a discontinuous nature at the end of the sample interval. This explains why the FFT is smeared over the frequency bandwidth. Basically, there are many terms needed in order to approximate this *apparently discontinuous* signal.

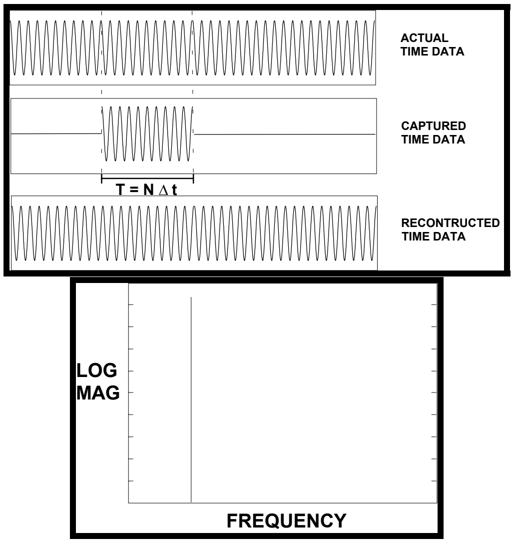

Figure 1

In order to minimize this error (and notice that I said *minimize* and not eliminate), we use weighting functions called *windows*. Basically we apply a weighting function to make the signal *appear* to better satisfy the periodicity requirements of the FFT process. Figure 3 shows a windowed time history.

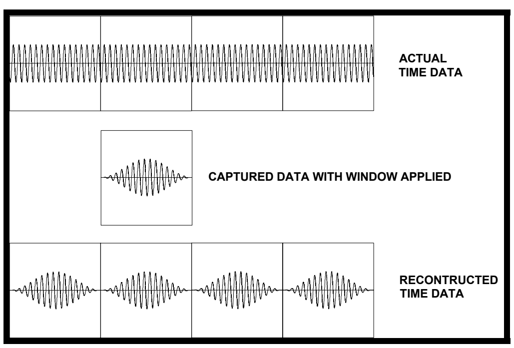

Figure 3

The most common windows for modal testing today are the Rectangular window, the Hanning window, and the Flat Top window for shaker testing and the Force/Exponential window for impact testing. The main thing to understand right from the start is that ***all windows distort data!*** Without going into all the detail, windows always distort the peak amplitude measured and always give the appearance of more damping then what actually exists in the measured FRF - two very important properties that we try to estimate from measured functions. The amplitudes are distorted as much as 36% for the Rectangular window and 16% for the Hanning window. The effects of these windows is best seen in the Frequency domain representation of the weighting function. All windows have a characteristic shape that identifies the amount of amplitude distortion possible, the damping effects introduced and the amount of smearing of information possible.
Figure 4 shows the Rectangular, Hanning and Flat Top windows frequency representation. Sometime soon we will discuss what

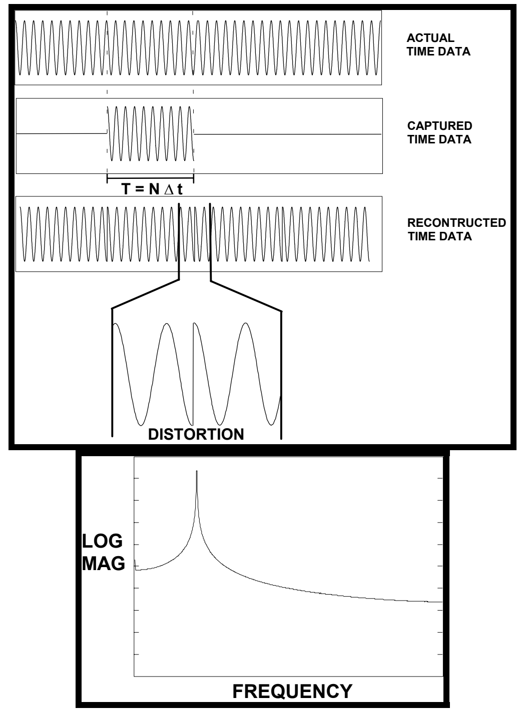

Figure 2

these curve more but for right now, I'm happy if you just understand that the windows, while a necessary evil in some measurement situations, distort data.

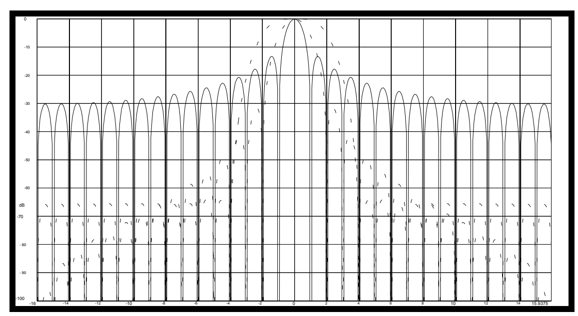
Figure 4

So how do I get around not using windows on measured FRFs for a modal test? Basically, I try to satisfy Fourier's request - "either sample a repetition of the data or completely observe the signal in one sample of data". If you think about it, signals such as pseudo-random, burst random, sine chirp, and digital stepped sine all satisfy this requirement under most conditions and therefore are leakage free and do not require the use of a window. Maybe we can discuss the particulars about each of the windows another time, but this short explanation should suffice for now.

Now I hope you understand why I don't like to use windows and I will avoid the use of windows at all costs - but every once and a while, I have no other choice. (Especially at home, where I can never get out of "doing windows"!) If you have any other questions about modal analysis, just ask me.